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Innovation, R&D spillovers, and the variety and concentration of local production structure

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Abstract

This paper presents a Cournot oligopoly model with R&D spillovers both within and across industries. The purpose is to give a proper theoretical foundation for the three different hypotheses regarding the impact of local production structure on innovation and output and address the mixed empirical results in this regard. Jacobs externalities are shown to have the firmest theoretical basis as both effective R&D and total industry output increase with the variety of industries. With respect to concentration, the outcome is more ambiguous, however, and depends on variety, both spillover rates, and R&D efficiency. If variety is limited then partial support is given to both Marshall-Arrow-Romer externalities, in the case of effective R&D, and Porter externalities, in the case of total industry output.

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1 Introduction

Since Glaeser et al. (1992), numerous empirical papers have attempted to uncover which local production structures, in terms of their variety and concentration, are the most conducive to innovation. That is, whether it is diversity or specialisation and likewise competition or concentration that make regional economies most innovative. However, the empirical results have been mixed and the three hypotheses in this regard, MAR, Jacobs, and Porter externalities, have all found some support (Beaudry and Schiffauerova, 2009; De Groot et al., 2009).

A further difficulty in making sense of the mixed findings is that neither Glaeser et al. (1992) nor the subsequent literature have provided any fully developed theories of how local production structure affects innovation. Theoretical foundations can be sought from industrial economics where there is now a considerable theoretical literature on knowledge spillovers and innovation incentives. Almost all of the literature, however, is concentrated on R&D spillovers within a single industry and how R&D cooperation can affect the outcome in this context (see, De Bondt, 1997).

It seems that only Steurs (1995) has considered the simultaneous occurrence of intra- and inter-industry knowledge spillovers so far. He examines the case of a two-industry, two-firm-per-industry model allowing for R&D spillovers to occur both within and between industries. Steurs' theoretical model demonstrates that the two spillover channels have different and yet interdependent effects on R&D, but leaves unanswered how local production structure affects the outcome. As noted by several authors, among the microfoundations of urban agglomeration, learning and knowledge spillovers are the least understood and advancing theoretical research on localised knowledge spillovers, which informs empirical research rather than lags behind it, is of urgent necessity (Duranton and Puga, 2004; Fujita and Krugman, 2004; Puga, 2010).

This paper builds on the previous theoretical research and consider the case of non-cooperative Cournot oligopolists that invest in cost-reducing or demand-enhancing technology. Steurs' (1995) model is extended to consider several industries as well as several firms within each of them. Hence, the firms make their R&D investments in the presence of spillovers both within and beyond their respective industries. The paper's theoretical contribution will be to show how the level of concentration within the local industries as well as the variety of these industries affects innovation and output. As such, it attempts to uncover the theoretical circumstances under which the three aforementioned hypotheses can be expected to hold. This will hopefully bring some clarity to the mixed empirical results and the overall implications for regional economic policy. The results show that variety increases both effective R&D and industry output, which is aligned with Jacobs externalities, and how the effect of concentration depends on several factors. In specific circumstances partial support is given to both MAR and Porter externalities, which also indicates how the choice of the variables can affect the empirical results.

The structure of the paper is as follows. The next section provides a brief review of both theoretical and empirical literature on inter- and intra-industry knowledge spillovers. The subsequent section presents our theoretical model and its equilibrium analysis. The most interesting questions concern the comparative statics of this equilibrium, in particular how effective R&D and total industry output respond to changes in the variety and concentration of local industries. These will be dealt with in a separate section followed by the conclusions.

2 Literature Review

To our knowledge, Bernstein (1988) is the first empirical study on both intra- and inter-industry R&D spillovers that considered these as proper externalities. Bernstein found that the firms' R&D investments were either substitutes for or complementary to intra-industry spillovers depending on the size of the firms' R&D propensities. The effect of inter-industry spillovers was found to be small and the same for all industries. Thus, the extent of intra-industry spillovers was the key factor according to Bernstein (1988).

Notwithstanding Bernstein's (1988) contribution, Glaeser et al. (1992) truly marked the beginning of a whole industry of empirical research that studied the relative importance of intra- and inter-industry spillovers and the role of local production structures in this respect. Based on the previous theoretical research, they formulated three distinct, but not necessarily mutually exclusive, hypotheses on localised knowledge spillovers, which were meant to explain the geographic concentration of R&D. The first one, MAR externalities, is attributed to the insights of the economists Alfred Marshall, Kenneth Arrow, and Paul Romer. According to this hypothesis, regions or cities that are specialised in one or few lines of industries have an advantage in the presence of localised knowledge spillovers and concentration of these industries is another conducive factor. Porter externalities, attributed to the management scholar Michael Porter, likewise flourish in specialised local production structures but conversely benefit from increased levels of competition. Lastly, Jacobs externalities, named after urban theorist Jane Jacobs, are most prominent in locations with diverse industrial base. Both variety of industries and the level of competition within them was considered to be conducive to innovation in this hypothesis. Interestingly, Glaeser et al. (1992) did not present any hypotheses according to which diverse *and* concentrated local production structures would be the best.

Glaeser et al. (1992) proceeded to test the hypotheses with U.S. city-level data and found support for Jacobs externalities. Numerous other scholars followed their suit and there has been a continuous stream of related empirical research to this date. In their recent survey, Beaudry and Schiffauerova (2009), for example, go through the results of 67 such studies. Similarly, De Groot et al. (2009) perform a meta-analysis of 31 empirical studies on agglomeration externalities. This takes us to the first major problem that this literature has faced: the empirical results are very mixed. Depending on the context (e.g. data, time period and the empirical model), all of the three hypotheses have found support but none of them in every case. The choice of the dependent variable, among others, has a tendency to lead to different results. An underlying problem, as several authors note (e.g. Breschi and Lissoni, 2001), is that these studies do not give any direct evidence of the existence of spillovers. The econometric research has regressed several performance measures against local attributes, assuming that knowledge spillovers explain the possible correlation between them.

The second major problem with this empirical research is that it does not have any robust theoretical basis. Glaeser et al. (1992) formulated their hypotheses loosely based on the ideas of the aforementioned scholars and neither they nor any subsequent writers, as far as we know, developed anything close to a formal model that could give insights to the underlying causal relationships. As a consequence, it is difficult to make sense of the mixed empirical findings. Without a formal theory, we have no clear idea how knowledge spillovers and local production structure affect firms' R&D incentives and what is then the outcome in terms of overall level of innovation.

Since the geography of innovation literature is very empirically orientated, the theoretical basis must be sought elsewhere. In industrial economics, following the seminal contributions of Spence (1984), d'Aspremont and Jacquemin (1988), and Kamien et al. (1992), there has been substantial theoretical research on (intra-industry) knowledge spillovers and innovation incentives. Two main ways of modelling R&D spillovers exist in this literature: the spillovers concern either R&D inputs or outputs. In both cases spillovers decrease firms' R&D investments: there is less to gain from these investments and more from free riding. The firms' effective R&D, which is the sum of its own R&D propensity and the received spillovers, however, is similarly decreasing with the spillover rate in the case of input spillovers. As such, input spillovers are an unlikely explanation for geographically concentrated innovation (Leppälä, 2014). However, in the case of output spillovers the effective R&D is maximised with spillover rate of exactly one half.

Relevant regarding our purposes is the finding by De Bondt et al. (1992) that in the case of a single industry effective R&D is increasing in concentration. If the number of firms in the industry increases, the firms have a less of an incentive in invest in R&D and this is not compensated by the increased number of spillover sources. Interestingly, the presence of inter-industry R&D spillovers affects this result as we will see.

A related strand of literature has considered whether localised knowledge spillovers lead to agglomeration (e.g. Van Long and Soubeyran, 1998). While the results are slightly mixed, Leppälä (2014) has shown that in the absence of any opposing factors firms in an industry would prefer to agglomerate in order to maximise the spillovers between them. Since inter-industry spillovers can be expected to only reinforce this incentive, we do not consider the location choice in this paper and assume that the firms and industries are agglomerated.

Even in industrial economics, research on inter-industry spillovers is almost non-existent and most papers are concerned with R&D cooperation in the context of a single industry. Extra-industry sources of R&D were already included in the model by Cohen and Levinthal (1989) but taken as exogenous. Later, Katsoutacos and Ulph (1998) and Leahy and Neary (1999) contrasted cases where the firms operate either in the same or different industries, but did not study the case where both inter- and intraindustry spillovers take place simultaneously. There are also some theoretical papers that consider spillovers between vertically related firms (Atallah, 2002; Ishii, 2004).

To our knowledge, the only paper that considers spillovers between completely segmented industries as well as within them is Steurs (1995). Steurs' model considers the case of two industries and two firms within each. Otherwise the firms and industries are completely identical, but the inter- and intra-industry spillover rates are allowed to be different. Steurs shows that intra-industry spillovers always increase effective R&D, since the strategic effect is missing. However, they also decrease the rate of intra-industry spillovers that maximises effective R&D. As such, inter- and intra-industry spillovers are interdependent. To analyse the effect of variety and concentration of local production structure, this paper extends Steurs' model to consider several firms as well as industries. R&D cooperation is not of direct interest in this case as it is not assumed by the three hypotheses and there is little empirical evidence of it (Brenner, 2007). As such, instead of addressing R&D cooperation in this paper we direct the reader to Steurs (1995) in this respect.

3 The model

We consider an agglomeration of m identical industries, each consisting of n identical firms that produce a homogeneous output. q_{ij} is the output produced and sold by firm i in industry j, and the total output of industry j is $Q_j = \sum_{i=1}^{n} q_{ij}$. It is further assumed that the markets are perfectly segmented and the final outputs are independent. In each market, the firms face a linear inverse demand curve, which shares the same characteristics: $P_j = a - Q_j \, \forall j \in m$, $a > Q_j \geq 0$.

The unit cost of firm i in industry j is $c_{ij} = c - X_{ij}$, where c is the initial marginal cost, common to all firms in each industry. X_{ij} is the firm's effective R&D, and $a > c > X_{ij} \geq 0$. The R&D output can be considered to be a cost reduction or equally well a demand-enhancing invention (De Bondt and Veugelers, 1991). Effective R&D is given by

$$X_{ij} = x_{ij} + \beta \sum x_{kj} + Z,$$

where x_{ij} is the firm's own R&D output, $\beta \sum x_{kj}$, $k \neq i$ are the output spillovers from the other firms in the same industry, and Z are the spillovers from firms in the other industries. $\beta \in [0, 1]$ is the intra-industry spillover rate. Furthermore, $Z = \sigma \sum\sum x_{il}$, $l \neq j$, where $\sigma \in [0, 1]$ is the inter-industry spillover rate.

As is often assumed in the literature, the cost of the firm's own R&D output x_{ij} is quadratic and given by

$$C(x_{ij}) = \frac{1}{2}\gamma x_{ij}^2,$$

where $\gamma > 0$ is an inverse measure of the efficiency of R&D. We assume the values of β and σ to be exogenous and reflect the extent to which R&D is leaked and useful across firms and different industries. This further assumes that the m industries are technologically related, such that some beneficial spillovers exist between them (Frenken et al., 2007). As already Bernstein and Nadiri (1988) showed, the set of industries bind by spillovers may not be large. As such, there may be other industries in the same location as well, but which are not related in this sense. It is a very stylised assumption that the firms and, in particular, industries are identical in every respect, but it facilitates our analysis with respect to the impact of variety and concentration. Later, we will shortly consider how some differences between the industries would affect the outcome.

The firms play a two-stage game. In the first stage, the firms in all industries decide simultaneously their R&D outputs, x_{ij}. In the second stage, the firms choose their final good outputs, q_{ij}, through Cournot competition. For expository reasons, we assume that there is no uncertainty with respect to the R&D output and discounting between the stages is also ignored. We derive the subgame perfect Nash equilibria by backward induction.

3.1 Symmetric equilibria

In the final production stage, firm i in industry j maximises its profit function given by

$$\pi_{ij} = (a - Q_j - c_{ij})q_{ij}.$$

The Cournot equilibrium output is

$$q_{ij}^* = \frac{a - nc_{ij} + \sum c_{kj}}{n+1} = \frac{a - c + Z + (n - (n-1)\beta)x_{ij} + (2\beta - 1)\sum x_{kj}}{n+1} \tag{1}$$

for all firms $i \in n, k \neq i$. Subsequently, the total output in industry j is

$$Q_j = \frac{n(a - c + Z) + (1 + \beta(n-1))\sum x_{ij}}{n+1}. \tag{2}$$

In the first stage, the firms choose their R&D levels. Given the subsequent output levels, firm i chooses x_{ij} in order to maximise

$$\pi_{ij} = (q_{ij}^*)^2 - \frac{1}{2}\gamma x_{ij}^2,$$

where q_{ij}^* is given by equation (1).

the first order condition gives the best-response function

$$x_{ij}(x_{kj}) = \frac{2(a - c + Z + (2\beta - 1)\sum x_{kj})(n - (n-1)\beta)}{\gamma(n+1)^2 - 2(n - (n-1)\beta)^2} \tag{3}$$

for firm i. This shows us that the R&D outputs x_{kj} are strategic substitutes to x_{ij} if $\beta < 1/2$ and complements if the inequality is reversed. Inter-industry spillovers, through Z, however, are always strategic complements.

The second order conditions in the R&D stage require that the numerator in the best response functions is positive. This holds for all $\beta \in [0, 1]$ when $\gamma > 2n^2/(n+1)^2$. The stability condition requires that the best response functions cross correctly (Henriques, 1990), and this holds $\forall \beta \in [0, 1]$ when $\gamma > 2n/(n+1)$.

Assuming first that the firms in industry j make a symmetric choice, $x_{ij} = x_j, \forall i \in n$, the best response function (3) gives the following equilibrium R&D output:

$$x_j^* = \frac{2(a-c+Z)(n-(n-1)\beta)}{\gamma(n+1)^2 - 2(n-(n-1)\beta)(1+(n-1)\beta)}. \tag{4}$$

While we will assuming symmetry also across the industries, equation (4) allows us to consider what implications any differences between them would have. Larger initial market size, $a-c$, or high R&D efficiency, i.e. lower γ, for example, would imply larger equilibrium R&D outputs in indystry j. Similarly, through Z, larger R&D outputs in other industries or a higher rate of inter-industry spillovers, σ, have the same effect. As such, there is a feedback loop through inter-industry spillovers that further reinforces any positive or negative effects. Döring and Schnellenbach (2006) note that it has been found that inter-industry spillovers tend to be asymmetric or one-directional. While our model does not consider such cases, we can conjecture that these would diminish the feedback loop.

Finally, we assume that the industries are identical and hence $x_j^* = x^*, \forall j \in m$. Substituting $Z = \sigma(m-1)nx^*$ into equation (4) gives the equilibrium R&D output:

$$x^* = \frac{2(a-c)(n-(n-1)\beta)}{\gamma(n+1)^2 - 2(n-(n-1)\beta)(n\sigma(m-1)+(n-1)\beta+1)}. \tag{5}$$

Interior and positive solutions for R&D outputs are guaranteed for $\gamma > 2n(nm-n+1)/(n+1)^2, \forall\beta, \sigma \in [0,1]$. We make the following assumption:

Assumption 1 $\gamma > \frac{2n(nm-n+1)}{(n+1)^2}$ *if* $m \geq 2$ *and* $\gamma > 2n/(n+1)$ *if* $m = 1$.

Besides guaranteeing the interior and positive solutions, this assumption then ensures that the stability conditions are met also in the case of a single industry. As such, we need to ensure that the R&D efficiency is not too high with respect to m and n. The assumption also indicates that $\gamma > 1$ by the very least.

Finally, it follows that effective R&D is given by

$$X = \frac{2(a-c)(n-(n-1)\beta)(n\sigma(m-1)+(n-1)\beta+1)}{\gamma(n+1)^2 - 2(n-(n-1)\beta)(n\sigma(m-1)+(n-1)\beta+1)}. \qquad (6)$$

4 Comparative statics

Instead of complete welfare analysis, this section concentrates on studying the changes in effective R&D and total industry output. These two are the most relevant concerning the independent variables used in the empirical literature, which are typically some measures of innovation, economic growth, or productivity (Beaudry and Schiffauerova, 2009). The first interesting issue regarding comparative statics concerns the effect that the two spillover rates have on effective R&D.

Proposition 1 *Inter-industry spillovers, σ, always increase effective R&D, whereas regarding intra-industry spillovers effective R&D is maximised for $\beta^* = \max\{\frac{1}{2}\frac{n-1-n\sigma(m-1)}{n-1}, 0\}$.*

Proof. Both β and σ exists only in the term

$$A \equiv (n-(n-1)\beta)(n\sigma(m-1)+(n-1)\beta+1)$$

in the numerator and denominator of equation (6), which is increasing in A. Since $\frac{\partial A}{\partial \sigma} = (n-(n-1)\beta)n(m-1) > 0$, also $\frac{\partial X}{\partial \sigma} = \frac{\partial X}{\partial A}\frac{\partial A}{\partial \sigma} > 0$.

Setting $\frac{\partial A}{\partial \beta} = -(n-1)(n\sigma(m-1)+(n-1)\beta)+1)+(n-(n-1)\beta)(n-1)$ equal to zero, gives the optimal intra-industry spillover rate $\beta^* = \frac{1}{2}\frac{n-1-n\sigma(m-1)}{n-1}$. Since the spillovers cannot be negative, $\beta^* = 0$ if $n\sigma(m-1) > n-1$. ■

As already noted by Steurs (1995), inter-industry spillovers reinforce the disincentive effect of intra-industry spillovers. However, and as can be expected, this does not only depend on the inter-industry spillover rate σ but also on the number of industries and firms. It is easy to verify that without inter-industry spillovers, the optimal intra-industry spillover rate corresponds to what has been found in the earlier literature, i.e. $\beta = 1/2$. However, the optimal β approaches zero as the inter-industry spillovers, through σ or m, increase, but an increase in the number firms works in the other direction as $\frac{\partial \beta^*}{\partial n} = \frac{1}{2}\frac{\sigma(m-1)}{(n-1)^2} \geq 0$.

This may seem to give some support to MAR and Porter externalities as the variety of sectors limits intra-industry spillovers. However, it is still the case that the maximal inter-industry spillover rate, $\sigma = 1$, and no intra-industry spillovers, $\beta = 0$, yield the highest effective R&D. It might be unrealistic to assume that the spillover rates differ from each other so much, however, especially that the inter-industry spillover rate is (substantially) higher than the intra-industry rate. For example, we might think that it is unlikely that firms could benefit more from external R&D that comes from a different industry than their own. As such, we analyse this trade of between the spillover rates by studying the case where the two rates are the same.

Proposition 2 *If intra- and inter-industry spillover rate are equal, $\beta = \sigma = \phi$, then the common spillover rate that maximises effective R&D is given by $\phi^* = \frac{1}{2}\frac{n^2m-2n+1}{n^2m-nm-n+1} \in \left[\frac{1}{2}, 1\right)$, with $\frac{\partial \phi^*}{\partial m} > 0$, $\frac{\partial \phi^*}{\partial n} < 0$.*

Proof. By setting $\beta = \sigma = \phi$, equation (6) becomes

$$X' = \frac{2(a-c)(n-(n-1)\phi)(nm\phi - \phi + 1)}{\gamma(n+1)^2 - 2(n-(n-1)\phi)(nm\phi - \phi + 1)}.$$

The first order condition,

$$\frac{\partial X'}{\partial \phi} = \frac{2(a-c)\gamma(n+1)^2(n^2m - 2n^2m\phi + 2nm\phi + 2n\phi - 2n - 2\phi + 1)}{(\gamma(n+1)^2 - 2(n-(n-1)\phi)(nm\phi - \phi + 1))^2} = 0,$$

gives the optimal spillover rate $\phi^* = \frac{1}{2}\frac{n^2m-2n+1}{n^2m-nm-n+1}$. ϕ^* is increasing in m, as $\frac{\partial \phi^*}{\partial m} = \frac{1}{2}\frac{n}{(nm-1)^2} > 0$, and decreasing in n, as $\frac{\partial \phi^*}{\partial n} = -\frac{1}{2}\frac{(m-1)(n^2m-1)}{(n-1)^2(nm-1)^2} < 0$.

Both inter- and intra-industry spillovers exist only when $m, n \geq 2$. Using L'Hôpital's rule, the lower bound of ϕ^* is given by $\lim_{n\to\infty} \phi^* = \frac{1}{2} = \underline{\phi}^*$. Using the same rule again, $\lim_{m\to\infty} \phi^* = \frac{1}{2}\frac{n}{n-1}$ at $n = 2$ gives the upper bound, $\bar{\phi}^* = 1$. ■

It is of some interest that the optimal spillover rate in this case is on the higher range of spillovers, bounded below by $\frac{1}{2}$. Not surprisingly, the optimal rate approaches 1 as variety increases, since this makes inter-industry spillovers relatively more important. The number of firms has the opposite effect, since the rivalry becomes more fierce.

Proposition 3 *An increase in the number of industries, m, always leads to an increase in effective R&D.*

Proof. This proof is similar to that of Proposition 1. m exists only in the term A of equation (6). It is straightforward to see that A is increasing in m and hence is also X. ■

This proposition gives the central support for Jacobs externalities as there is no tradeoff with respect to variety. Such a tradeoff may arise if space is limited, but it also requires that the number of firms within an industry has a similar effect. This another aspect of Jacobs externalities, which is the positive effect of competition, will be studied next.

Proposition 4 *If $3\beta\sigma m + 4\beta^2 - 3\beta\sigma - 2\sigma m - 4\beta + 2\sigma + 1 > 0$, which is always the case if $\beta \geq \frac{2}{3}$ or either $\sigma = 0$ or $m = 1$ and $\beta \neq \frac{1}{2}$, effective R&D is maximised for $n^* = \frac{\beta\sigma m + 4\beta^2 - \beta\sigma - 4\beta + 1}{3\beta\sigma m + 4\beta^2 - 3\beta\sigma - 2\sigma m - 4\beta + 2\sigma + 1}$ firms, where $\frac{\partial n^*}{\partial m}, \frac{\partial n^*}{\partial \sigma} > 0$, and $\frac{\partial n^*}{\partial \beta} \geq 0$ if $\beta \leq 1 - \frac{1}{2}\sqrt{\sigma m - \sigma + 1}$. If $3\beta\sigma m + 4\beta^2 - 3\beta\sigma - 2\sigma m - 4\beta + 2\sigma + 1 \leq 0$, then effective R&D is always increasing in n.*

Proof. Effective R&D is non-decreasing in n when

$$\frac{\partial X}{\partial n} = \frac{2(a-c)\gamma(n+1)^2(Cn - B)}{(\gamma(n+1)^2 - 2(n - (n-1)\phi)(nm\phi - \phi + 1))^2} \geq 0, \qquad (7)$$

with

$$B = -\beta\sigma m - 4\beta^2 + \beta\sigma + 4\beta - 1,$$

$$C = -3\beta\sigma m - 4\beta^2 + 3\beta\sigma + 2\sigma m + 4\beta - 2\sigma - 1.$$

Since the other terms in equation (7) are always positive, its sign depends on the sign of $(Cn - B)$. Hence, effective R&D is increasing in n when $Cn \geq B$. B is non-increasing in m and σ, since $\frac{\partial B}{\partial m} = -\beta\sigma \leq 0$ and $\frac{\partial B}{\partial \sigma} = -\beta m + \beta \leq 0$. When $m = 1$ or $\sigma = 0$, $B = -4\beta^2 + 4\beta - 1 \leq 0$, which holds as an equality when $\beta = \frac{1}{2}$. Therefore, $B \leq 0$.

If $C < 0$, effective R&D is non-decreasing in n when

$$n \leq \frac{B}{C} = \frac{\beta\sigma m + 4\beta^2 - \beta\sigma - 4\beta + 1}{3\beta\sigma m + 4\beta^2 - 3\beta\sigma - 2\sigma m - 4\beta + 2\sigma + 1}, \tag{8}$$

and maximised when (8) holds as an equality. n^* is increasing in m and σ, since

$$\frac{\partial n^*}{\partial m} = \frac{2\sigma(1-\beta)(2\beta-1)^2}{(3\beta\sigma m + 4\beta^2 - 3\beta\sigma - 2\sigma m - 4\beta + 2\sigma + 1)^2} > 0$$

and

$$\frac{\partial n^*}{\partial \sigma} = \frac{2(m-1)(1-\beta)(2\beta-1)^2}{(3\beta\sigma m + 4\beta^2 - 3\beta\sigma - 2\sigma m - 4\beta + 2\sigma + 1)^2} > 0.$$

n^* is also non-decreasing in β when

$$\frac{\partial n^*}{\partial \beta} = \frac{2\sigma(m-1)(4\beta^2 - \sigma m - 8\beta + \sigma + 3)}{(3\beta\sigma m + 4\beta^2 - 3\beta\sigma - 2\sigma m - 4\beta + 2\sigma + 1)^2} \geq 0$$

or

$$4\beta^2 - \sigma m - 8\beta + \sigma + 3 \geq 0. \tag{9}$$

Equation (9) has two roots, $\beta = 1 \pm \frac{1}{2}\sqrt{\sigma m - \sigma + 1}$. Since the larger root is more than 1 and the leading coefficient positive, equation (9) is non-negative when $\beta \leq 1 - \frac{1}{2}\sqrt{\sigma m - \sigma + 1}$.

C is decreasing in β when $\frac{\partial C}{\partial \beta} = -3\sigma m - 8\beta + 3\sigma + 4 < 0$ or $\beta > -\frac{3}{8}\sigma m + \frac{3}{8}\sigma + \frac{1}{2} \leq \frac{1}{2}$. When $\beta = \frac{2}{3}$, $C = -\frac{1}{9}$, and therefore $C < 0$ always when $\beta \geq \frac{2}{3}$. If $m = 1$ or $\sigma = 0$, then $C = -4\beta^2 + 4\beta - 1 < 0$ as long as $\beta \neq \frac{1}{2}$.

If $C > 0$, effective R&D is non-decreasing in n when $n \geq \frac{B}{C}$, which holds for all n since then $\frac{B}{C} \leq 0$. If $C = 0$, then equation (7) is always non-negative. ■

With respect to competition, the results are more mixed. They are aligned with Jacobs externalities in the sense that a wider variety makes the optimal number of firms for effective R&D larger or even infinite. However, if variety is low or intra-industry spillovers high then a more concentrated industry becomes optimal for effective R&D. The competitive effect in own industry dominates the spillovers received from the others. This issue will be studied more closely in the following corollary.

Corollary 1 *In the absence of inter-industry R&D spillovers a monopoly maximises effective R&D, expect when $\beta = \frac{1}{2}$, in which case the number of firms has no effect and $X = \frac{a-c}{2\gamma-1}$.*

Proof. If either $\sigma = 0$ or $m = 1$, then $B = C$. B and C are not equal to zero if $\beta \neq \frac{1}{2}$, and from equation (8) we get that effective R&D is maximised for $n = 1$. Equation (6) gives $X = \frac{a-c}{2\gamma-1}$ in this case.

If $\beta = 1/2$, then $B = C = 0$ and equation (7) is zero. Again, from equation (6) we get that $X = \frac{a-c}{2\gamma-1}$. ■

As De Bondt et al. (1992) already showed, we see from this result that effective R&D is typically decreasing with competition within a single industry. An interesting exception that was not mentioned by De Bondt et al. (1992), though, is $\beta = \frac{1}{2}$ when the number of firms has no effect. Nevertheless, this result partially supports MAR externalities. That is, if variety is low then concentration increases effective R&D. On the other hand, this result shows that effective R&D might not always be the key factor that we are interested in. Indeed, the dependent variable in the empirical research has neither always been a measure of R&D. Some researchers have studied the impact of variety and concentration on variables such as employment and output instead. As such, it is relevant to see how total industry output is affected, since this is more relevant with respect to some of the empirical studies.

Proposition 5 *Total industry output is increasing in m and σ, as well as in β when $\beta \leq \max\{\frac{1}{2}\frac{n-1-n\sigma(m-1)}{n-1}, 0\}$.*

Proof. Total output in a single industry is

$$Q = n\frac{a-c+X}{n+1}, \tag{10}$$

where X is given by equation (6). Since m, σ, and β appear only in X and Q is increasing in X, it must be that Q is always increasing in m and σ as well as in β when $\beta \leq \max\{\frac{1}{2}\frac{n-1-n\sigma(m-1)}{n-1}, 0\}$, as determined in Propositions 1 and 3 earlier. ■

Again, we are given further support for Jacobs spillover hypothesis as also total industry output is increasing in variety of industries as well as spillovers from them. Indeed, also in empirical research evidence of Jacobs externalities is most often found when studying economic growth (Beaudry and Schiffauerova, 2009). As in Proposition 1, optimal intra-industry spillover rate is shown to be restricted by inter-industry spillovers. The final step is to see how the concentration of industries affects total industry output.

Proposition 6 *Total industry output is increasing in n if $4\beta\sigma m + 6\beta^2 - 4\beta\sigma - 2\sigma m - 6\beta + 2\sigma + 2 - \gamma \leq 0$, which is always the case if $\beta \leq \frac{1}{2}$ and $\gamma \geq 2\sigma - 2\sigma m + 2$. If $4\beta\sigma m + 6\beta^2 - 4\beta\sigma - 2\sigma m - 6\beta + 2\sigma + 2 - \gamma > 0$, total industry output is maximised for $n^* = \frac{2(2\beta^2-2\beta+\gamma)}{4\beta\sigma m+6\beta^2-4\beta\sigma-2\sigma m-6\beta+2\sigma+2-\gamma}$ firms, where $\frac{\partial n^*}{\partial m}, \frac{\partial n^*}{\partial \sigma} < 0$, and $\frac{\partial n^*}{\partial \gamma} > 0$ if $\beta > \frac{1}{2}$.*

Proof. An increase in n changes total industry output by

$$Q(n+1) - Q(n) = \frac{(a-c)(n+1)n\gamma}{\gamma(n+1)^2 - 2(n-(n-1)\beta)(n\sigma(m-1)+(n-1)\beta+1)}$$

$$-\frac{(a-c)(n+2)(n+1)\gamma}{\gamma(n+2)^2-2(n-n\beta+1)(n\sigma(m-1)+n\beta+\sigma m-\sigma+1)}$$

or

$$Q(n+1)-Q(n)=\frac{(a-c)(n+1)\gamma(Dn-E)}{F}, \tag{11}$$

with

$$D=-4\beta\sigma m-6\beta^2+4\beta\sigma+2\sigma m+6\beta-2\sigma-2+\gamma,$$

$$E=-2(2\beta^2-2\beta+\gamma),$$

$$F=(\gamma(n+1)^2-2(n-(n-1)\beta)(n\sigma(m-1)+(n-1)\beta+1))$$
$$\times(\gamma(n+2)^2-2(n-n\beta+1)(n\sigma(m-1)+n\beta+\sigma m-\sigma+1)).$$

Given the positivity of outputs, $F>0$. Hence, the sign of equation (11) depends on the sign of $(Dn-E)$. Given Assumption 1, also $E<0$. Therefore, if $D\geq 0$, output is increasing in n. D is decreasing in β when $\frac{\partial D}{\partial\beta}=-4\sigma m-12\beta+4\sigma+6<0$ or $\beta>-\frac{1}{3}\sigma m+\frac{1}{3}\sigma+\frac{1}{2}\leq\frac{1}{2}$. When $\beta=\frac{1}{2}$, $D=\gamma-\frac{1}{2}$, which is positive given Assumption 1. When $\beta=0$, $D=\gamma-2\sigma+2\sigma m-2$, and therefore $D\geq 0$ if both $\beta\leq\frac{1}{2}$ and $\gamma\geq 2\sigma-2\sigma m+2$.

If $D<0$, industry output is maximised for

$$n^*=\frac{E}{D}=\frac{2(2\beta^2-2\beta+\gamma)}{4\beta\sigma m+6\beta^2-4\beta\sigma-2\sigma m-6\beta+2\sigma+2-\gamma}.$$

If $\beta\geq\frac{1}{2}$, then

$$\frac{\partial n^*}{\partial m}=-\frac{2(2\beta^2-2\beta+\gamma)(4\beta\sigma-2\sigma)}{(4\beta\sigma m+6\beta^2-4\beta\sigma-2\sigma m-6\beta+2\sigma+2-\gamma)^2}<0,$$

$$\frac{\partial n^*}{\partial\sigma}=-\frac{2(2\beta^2-2\beta+\gamma)(4\beta m-4\beta-2m+2))}{(4\beta\sigma m+6\beta^2-4\beta\sigma-2\sigma m-6\beta+2\sigma+2-\gamma)^2}<0,$$

and

$$\frac{\partial n^*}{\partial\gamma}=\frac{4(2\beta-1)(\sigma m+2\beta-\sigma-1)}{(4\beta\sigma m+6\beta^2-4\beta\sigma-2\sigma m-6\beta+2\sigma+2-\gamma)^2}>0$$

show how n^* responds to changes in m, σ, and γ. ■

Like in the case of effective R&D, whether total industry output is increasing or decreasing with concentration depends on both spillover rates, variety of industries, and now also on R&D efficiency. Comparing Propositions 4 and 6, we see that the conditions for both when n has always a positive effect as well as optimal, finite n, when it exist, are different. As such, there can be cases where concentration affects positively effective R&D but not output. From empirical literature it can also be seen that when the independent variable is economic growth, MAR externalities appear to have far less often a positive impact and in several cases a negative one (Beaudry and Schiffauerova, 2009). This issue is most clearly illustrated by examining the case where there are no inter-industry spillovers (or they are held constant):

Corollary 2 *In the absence of inter-industry R&D spillovers, total industry output is always increasing in n when $\gamma \geq 6\beta^2 - 6\beta + 2$ and never maximised by monopoly.*

Proof. When $m = 1$, $D = -6\beta^2 + 6\beta + \gamma - 2$, which is non-negative when $\gamma \geq 6\beta^2 - 6\beta + 2$. If monopoly maximises total industry output, then

$$n^* = \frac{2(2\beta^2 - 2\beta + \gamma)}{6\beta^2 - 6\beta - \gamma + 2} < 2$$

or $\gamma < 2\beta^2 - 2\beta + 1 \leq 1$, but this contradicts Assumption 1. ■

Corollary 2 shows that there exists a wide range of cases where total industry output, when studying a single industry, is increasing with the number firms. This holds, for example, when β is $\frac{1}{2}$ or not too far from it or always when $\gamma > 2$. As such, this time the model gives partial support to Porter externalities as with limited variety competition improves the outcome in terms of output. A further contrast with Corollary 1 is that in this case monopoly is never the optimal market structure. As such, it is not surprising that even when monopoly leads to the maximal effective R&D, monopoly output is not the largest possible. This is a further illustration of how the choice of the dependent variable in empirical research can have critical consequences.

5 Conclusion

The purpose of this paper was twofold. First, to give proper theoretical foundations for MAR, Porter, and Jacobs externalities. Second, address the mixed empirical results with respect to the three hypotheses. The theoretical model lends most support to Jacobs externalities, and this is aligned with the empirical results (Beaudry and Schiffauerova, 2009; De Groot et al., 2009). That is, both effective R&D and total industry output can be shown to be increasing with the variety of local industries. However, here we considered the number of industries between which there are spillovers, which is not necessarily the same as the total number of local industries. More recent empirical studies, following Frenken et al. (2007), have taken into account this ”related variety” of local industries.

With respect to concentration, the theoretical model gives more mixed implications as the outcome depends on the variety of industries, intra- and inter-industry spillover rates, and R&D efficiency. However, this may help to clarify similarly mixed empirical results (De Groot et al., 2009). If variety is high, effective R&D can be increasing in competition. However, this requires that intra-industry spillovers are not too high as otherwise there exists an optimal industry concentration. The outcome is similar with respect to total industry output, but there exists a wider variety of cases where output is increasing with competition or the optimal number of firms is higher. Nevertheless, the support for Jacobs externalities in this regard is partial. Why Glaeser et al. (1992) attributed competition to Jacobs externalities is less clear, because Jacobs (1969) merely argues that larger organisations tend to be less innovative (see, also Beaudry and Schiffauerova, 2009). Nevertheless, it is important to acknowledge that the effect of local production structure may depend on the used performance variable. If variety is low, then concentration is found to increase effective R&D, which then gives partial support for the MAR externalities. On the other hand, when variety is low, competition can typically increases output, which then supports Porter externalities. When it comes empirical research, therefore, the choice of the dependent variable can be critical.

How the independent variables are chosen is a critical issue as well. For example, under the same circumstances effective R&D can be shown to be increasing with the average number of firms but to decrease if only the number of firms in that particular industry increases. A related issue is that many empirical studies have used relative measures for variety and concentration, such as the Herfindahl-Hirschman Index. Since the theoretical model was based on absolute measures, it does not give direct explanation for why and how these relative measures should matter. Furthermore, the choice between relative and absolute measures tends to affect the outcome of the analysis substantially (De Groot et al., 2009).

While the model was based on the most standard spillover models in the literature, and hence a logical way to proceed, it is worth considering how well it corresponds to the empirical models and the reality they are trying to explain. One obvious and crude simplification is the assumed symmetry between firms and industries, but a few others are worth mentioning. The underlying idea of Porter externalities is that competition fosters innovation because otherwise the firms would not survive. Since this aspect of competition is missing from our model, it may not do full justice to Porter externalities. However, this shows that further theoretical work regarding Porter externalities is warranted. One way to extend the model would be to introduce absorptive capacity into it, as in Martin (2002), for example, since its relevance has been emphasised in the geography of innovation literature. However, while absorptive capacity has been found to increase the firms' R&D investments it is not clear that it would reverse the tendencies with respect to variety and concentration as uncovered here.

Another point worth emphasising is that some authors have not only considered spillovers in terms of imitating existing technologies but how they also foster subsequent inventions. This point was particularly relevant to Jacobs (1969). While it is not clear how this phenomenon should be formalised, possibly it could be the case that instead of providing direct inter-industry spillovers variety increases firms' R&D efficiency. Again, this would likely support our main finding that inter-industry R&D spillovers strongly facilitate innovation and growth. To sum up, while this paper calls for more theoretical work and is merely an early step in this regard, it also points to several critical issues that should be taken into account when building the empirical models for studying localised knowledge spillover and interpreting their results.

References

Atallah, G. (2002). Vertical R&D spillovers, cooperation, market structure, and innovation. *Economics of Innovation and New Technology*, 11(3):179–209.

Beaudry, C. and Schiffauerova, A. (2009). Who's right, Marshall or Jacobs? The localization versus urbanization debate. *Research Policy*, 38(2):318–337.

Bernstein, J. I. (1988). Costs of production, intra-and interindustry R&D spillovers: Canadian evidence. *Canadian Journal of Economics*, 21(2):324–47.

Bernstein, J. I. and Nadiri, M. I. (1988). Interindustry R&D spillovers, rates of return, and production in high-tech industries. *American Economic Review*, 78(2):429–34.

Brenner, T. (2007). Local knowledge resources and knowledge flows. *Industry and Innovation*, 14(2):121–128.

Breschi, S. and Lissoni, F. (2001). Knowledge spillovers and local innovation systems: a critical survey. *Industrial and Corporate Change*, 10(4):975–1005.

Cohen, W. M. and Levinthal, D. A. (1989). Innovation and learning: The two faces of R&D. *Economic Journal*, 99(397):569–96.

d'Aspremont, C. and Jacquemin, A. (1988). Cooperative and noncooperative R&D in duopoly with spillovers. *American Economic Review*, 78(5):1133–1137.

De Bondt, R. (1997). Spillovers and innovative activities. *International Journal of Industrial Organization*, 15(1):1–28.

De Bondt, R., Slaets, P., and Cassiman, B. (1992). The degree of spillovers and the number of rivals for maximum effective R&D. *International Journal of Industrial Organization*, 10(1):35–54.

De Bondt, R. and Veugelers, R. (1991). Strategic investment with spillovers. *European Journal of Political Economy*, 7(3):345–366.

De Groot, H. L., Poot, J., and Smit, M. J. (2009). Agglomeration externalities, innovation and regional growth: theoretical perspectives and meta-analysis. In Capello, R. and Nijkamp, P., editors, *Handbook of Regional Growth and Development Theories*, pages 256–281. Edward Elgar Publishing, Cheltenham.

Döring, T. and Schnellenbach, J. (2006). What do we know about geographical knowledge spillovers and regional growth?: a survey of the literature. *Regional Studies*, 40(3):375–395.

Duranton, G. and Puga, D. (2004). Micro-foundations of urban agglomeration economies. In Henderson, J. V. and Thisse, J.-F., editors, *Handbook of Regional and Urban Economics*, volume 4, pages 2063–2117. Elsevier.

Frenken, K., Van Oort, F., and Verburg, T. (2007). Related variety, unrelated variety and regional economic growth. *Regional studies*, 41(5):685–697.

Fujita, M. and Krugman, P. (2004). The new economic geography: Past, present and the future. *Papers in Regional Science*, 83(1):139–164.

Glaeser, E. L., Kallal, H. D., Scheinkman, J. A., and Shleifer, A. (1992). Growth in cities. *Journal of Political Economy*, 100(6):1126–1152.

Henriques, I. (1990). Cooperative and noncooperative R&D in duopoly with spillovers: Comment. *American Economic Review*, 80(3):638–40.

Ishii, A. (2004). Cooperative R&D between vertically related firms with spillovers. *International Journal of Industrial Organization*, 22(8):1213–1235.

Jacobs, J. (1969). *The Economy of Cities*. Random House.

Kamien, M. I., Muller, E., and Zang, I. (1992). Research joint ventures and R&D cartels. *American Economic Review*, 82(5):1293–1306.

Katsoutacos, Y. and Ulph, D. (1998). Endogenous spillovers and the performance of research joint ventures. *The Journal of Industrial Economics*, 46(3):333–357.

Leahy, D. and Neary, J. P. (1999). R&D spillovers and the case for industrial policy in an open economy. *Oxford Economic Papers*, 51(1):40–59.

Leppälä, S. (2014). Theoretical perspectives on localised knowledge spillovers and agglomeration. *Cardiff Economics Working Papers*, E2014/10.

Martin, S. (2002). Spillovers, appropriability, and R&D. *Journal of Economics*, 75(1):1–32.

Puga, D. (2010). The magnitude and causes of agglomeration economies. *Journal of Regional Science*, 50(1):203–219.

Spence, A. M. (1984). Cost reduction, competition, and industry performance. *Econometrica*, 52(1):101–21.

Steurs, G. (1995). Inter-industry R&D spillovers: what difference do they make? *International Journal of Industrial Organization*, 13(2):249–276.

Van Long, N. and Soubeyran, A. (1998). R&D spillovers and location choice under Cournot rivalry. *Pacific Economic Review*, 3(2):105–119.